

A certain economist believes that the rate at which a person's wealth changes is proportional to the difference between their country's median wealth and their own wealth. Assuming that median wealth is a constant ( $W_{\text{MEDIAN}}$ ), and that wealthy people (people with a lot of wealth) tend to get wealthier, write a differential equation for the wealth  $W(t)$  of a wealthy person at time  $t$ .

SCORE: \_\_\_\_ / 3 PTS

NOTE: The sign of all constants should be stated clearly.

$$\textcircled{2} \quad \frac{dW}{dt} = k(W - W_{\text{MEDIAN}}), \quad \underline{k > 0} \quad \textcircled{1}$$

(WEALTHY PERSON  $\rightarrow$  WEALTH INCREASES  $\rightarrow \frac{dW}{dt} > 0$   
ALSO,  $W > W_{\text{MEDIAN}} \rightarrow W - W_{\text{MEDIAN}} > 0$   
SO  $k > 0$ )

What does the Existence and Uniqueness Theorem tell you about possible solutions to the initial value problem

SCORE: \_\_\_\_ / 4 PTS

$(\frac{dy}{dx})^3 + 1 = y$ ,  $y(3) = 1$  ? Justify your answer properly, but briefly.

①  $\frac{dy}{dx} = (y-1)^{\frac{1}{3}} = f$

①  $f_y = \frac{1}{3}(y-1)^{-\frac{2}{3}}$

① IS NOT CONTINUOUS AROUND  $(3, 1)$   
SINCE  $f_y(3, 1)$  DNE

E+U TELLS US NOTHING (NO CONCLUSION)

①



WRONG IF YOU SAID

"THERE IS NO UNIQUE SOLUTION"

Consider the IVP  $y' = x(y-1)$ ,  $y(2) = 6$ . Use Euler's method with  $h = 0.1$  to estimate  $y(2.2)$ .

SCORE: \_\_\_\_ / 4 PTS

$$y(2.1) \approx y(2) + y'(2,6)(2.1-2) = 6 + 2(6-1)(0.1) = 7 \quad \textcircled{1}$$

$$y(2.2) \approx y(2.1) + y'(2.1,7)(2.2-2.1) \approx 7 + 2.1(7-1)(0.1) \quad \textcircled{1}$$
$$= 8.26 \quad \textcircled{1}$$

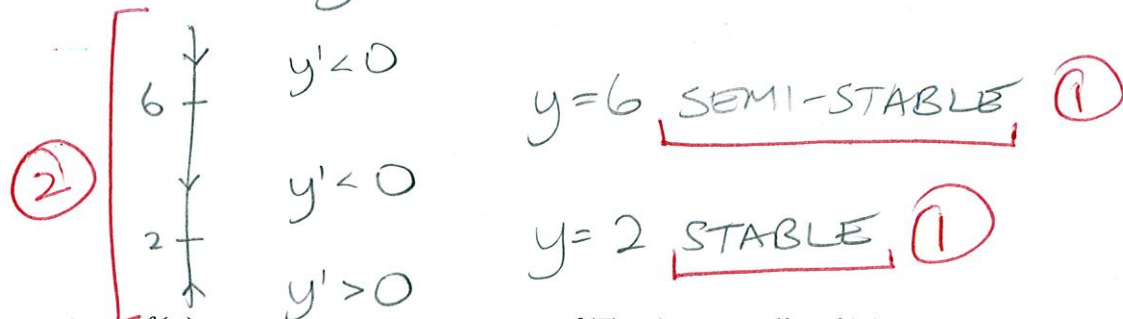
Consider the autonomous DE  $y' = (2 - y)^3(6 - y)^2$ .

SCORE: \_\_\_\_ / 6 PTS

- [a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.

You must draw a phase portrait to get full credit.

$$y' = 0 \rightarrow y = 2, 6$$



- [b] If  $y = f(x)$  is a solution of the DE such that  $f(7) = 1$ , what is  $\lim_{x \rightarrow \infty} f(x)$ ?

① 2

- [c] If  $y = g(x)$  is a solution of the DE such that  $g(8) = 5$ , what is  $\lim_{x \rightarrow \infty} g(x)$ ?

① 2

Consider the DE  $x^2 y'' - xy' + y = \sqrt{x}$ .

SCORE: \_\_\_\_ / 7 PTS

[a] Is  $y = 4\sqrt{x} + Ax + Bx \ln x$  a family of solutions of the DE?

$$y' = 2x^{-\frac{1}{2}} + A + B(1 + \ln x) \quad (1)$$

$$y'' = -x^{-\frac{3}{2}} + \frac{B}{x} \quad (1)$$

$$x^2 y'' - xy' + y = \begin{array}{l} -x^{\frac{1}{2}} + Bx \quad (1) \\ -2x^{\frac{1}{2}} - Ax - Bx - Bx \ln x \\ + 4x^{\frac{1}{2}} + Ax \quad \quad \quad + Bx \ln x \end{array}$$

$$= x^{\frac{1}{2}} \quad \text{YES} \quad (1)$$

[b] If the answer to [a] is "YES", solve the IVP consisting of the DE and the initial conditions  $y(1) = 6$ ,  $y'(1) = 2$ .

If the answer to [a] is "NO", write "SKIP" and skip this part.

$$y(1) = 4\sqrt{1} + A(1) + B(1)\ln 1 = 4 + A = 6 \rightarrow A = 2 \quad \left(\frac{1}{2}\right)$$

$$y'(1) = \frac{2}{\sqrt{1}} + A + B(1 + \ln 1) = 2 + 2 + B = 4 + B = 2 \quad \left(\frac{1}{2}\right)$$
$$B = -2$$

$$y = 4\sqrt{x} + 2x - 2x \ln x \quad (1)$$