A certain economist believes that the rate at which a person's wealth changes is proportional to the difference between their country's median wealth and their own wealth. Assuming that median wealth is a constant (W_{MEDIAN}), and that wealthy people (people with a lot of wealth) tend to get wealthier, write a differential equation for the wealth $W(t)$ of a wealthy person at time t. **SCORE: _____ / 3 PTS**

NOTE: The sign of all constants should be stated clearly.

2 $\dfrac{dW}{dt} = k(W - W_{MEDIAN})$, $k > 0$ 1

(WEALTHY PERSON $\longrightarrow$ WEALTH INCREASES $\longrightarrow \dfrac{dW}{dt} > 0$

ALSO, $W > W_{MEDIAN} \longrightarrow W - W_{MEDIAN} > 0$

SO $k > 0$)

What does the Existence and Uniqueness Theorem tell you about possible solutions to the initial value problem **SCORE: _____ / 4 PTS**

$(\frac{dy}{dx})^3 + 1 = y, \quad y(3) = 1$? **Justify your answer properly, but briefly.**

1 $\frac{dy}{dx} = (y-1)^{\frac{1}{3}} = f$

1 $f_y = \frac{1}{3}(y-1)^{-\frac{2}{3}}$ IS NOT CONTINUOUS AROUND $(3,1)$ 1

SINCE $f_y(3,1)$ DNE

E+U TELLS US NOTHING (NO CONCLUSION)

1

WRONG IF YOU SAID "THERE IS NO UNIQUE SOLUTION"

Consider the IVP $y' = x(y-1)$, $y(2) = 6$. Use Euler's method with $h = 0.1$ to estimate $y(2.2)$. **SCORE: _____ / 4 PTS**

$$y(2.1) \approx y(2) + y'(2,6)(2.1-2) = 6 + 2(6-1)(0.1) = 7 \quad 1 \quad 1$$

$$y(2.2) \approx y(2.1) + y'(2.1,7)(2.2-2.1) \approx 7 + 2.1(7-1)(0.1) \quad 1$$

$$= 8.26 \quad 1$$

Consider the autonomous DE $y' = (2-y)^3(6-y)^2$.

SCORE: _____ / 6 PTS

[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.
<u>**You must draw a phase portrait to get full credit.**</u>

$y' = 0 \rightarrow y = 2, 6$

2

$y' < 0$

$y' < 0$

$y' > 0$

$y = 6$ SEMI-STABLE 1

$y = 2$ STABLE 1

[b] If $y = f(x)$ is a solution of the DE such that $f(7) = 1$, what is $\lim_{x\to\infty} f(x)$?

1 2

[c] If $y = g(x)$ is a solution of the DE such that $g(8) = 5$, what is $\lim_{x\to\infty} g(x)$?

1 2

Consider the DE $x^2y'' - xy' + y = \sqrt{x}$.

SCORE: _____ / 7 PTS

[a] Is $y = 4\sqrt{x} + Ax + Bx\ln x$ a family of solutions of the DE ?

$y' = 2x^{-\frac{1}{2}} + A + B(1 + \ln x)$ (1)

$y'' = -x^{-\frac{3}{2}} + \frac{B}{x}$ (1)

$$x^2y'' - xy' + y = \boxed{\begin{array}{l} -x^{\frac{1}{2}} + \cancel{Bx} \\ -2x^{\frac{1}{2}} - \cancel{Ax} - \cancel{Bx} - \cancel{Bx\ln x} \\ +4x^{\frac{1}{2}} + \cancel{Ax} \qquad + \cancel{Bx\ln x} \end{array}}$$ (1)

$= x^{\frac{1}{2}}$, (1) YES, (1)

[b] If the answer to [a] is "YES", solve the IVP consisting of the DE and the initial conditions $y(1) = 6, \quad y'(1) = 2$.
If the answer to [a] is "NO", write "SKIP" and skip this part.

$y(1) = 4\sqrt{1} + A(1) + B(1)\ln 1 = 4 + A = 6 \rightarrow A = 2$ ($\frac{1}{2}$)

$y'(1) = \frac{2}{\sqrt{1}} + A + B(1 + \ln 1) = 2 + 2 + B = 4 + B = 2$ ($\frac{1}{2}$)

$B = -2$

$y = 4\sqrt{x} + 2x - 2x\ln x$ (1)